

## Threshold-Convergent Systems: The Shared Mathematical Structure Governing Quantum Error Correction and Oracle Consensus for Physical Asset Verification and Collateralisation Under Basel IV

A Formal Characterisation of Distributed Systems in Which Scaling Below a Critical Error Threshold Produces Exponential Reliability Improvement, with Applications to Tokenized Physical Asset Collateralisation Under Basel SCO60

### Abstract

This paper identifies and formally characterises a class of distributed information systems -- which we term threshold-convergent systems -- in which individual participants are unreliable, but a mathematically definable critical threshold exists such that when participant error rates fall below it, adding more participants produces exponential improvement in system-level reliability. Above the threshold, scale amplifies noise. Below it, scale suppresses noise exponentially. We establish four axiomatic properties that define the class: component unreliability, threshold existence as a phase boundary, emergent composability of reliable outputs from unreliable inputs, and adversarial resistance up to formally bounded fractions.

We demonstrate that this threshold phenomenon governs two independently developed systems operating in entirely different physical domains: quantum error correction, as demonstrated by Google Quantum AI's Willow processor achieving below-threshold surface code performance in December 2024, and oracle consensus for physical asset verification, as implemented in the CVR Protocol's reputation-weighted Bayesian fusion with Markov Chain Monte Carlo convergence guarantees. We derive the formal structural mapping between these two systems, establishing that the error suppression factor  $\gamma$  in quantum surface codes corresponds to the posterior credible interval narrowing rate in MCMC-convergent oracle networks, that qubit code distance scaling corresponds to oracle network scaling, and that the surface code error threshold corresponds to the oracle deviation threshold. We further connect the phase transition structure to the 2D random-bond Ising model mapping established by Dennis et al. for quantum error correction, and show that the MCMC ergodic regime transition in oracle consensus is governed by the same class of mathematical theorems about when distributed, noisy, stochastic processes produce reliable collective outputs.

We then demonstrate that this shared mathematical structure has direct regulatory implications: the threshold-convergent property of the CVR Protocol's oracle network is the precise mechanism that satisfies the Basel Committee on Banking Supervision's SCO60 'ongoing basis' classification requirement for Group 1a tokenized physical assets. The dynamic verification discount derived from the MCMC posterior credible interval provides continuously updating capital relief calculable from the threshold-convergent mathematics, and the framework enables a principled three-class regulatory taxonomy distinguishing threshold-convergent verification from non-convergent continuous monitoring and periodic audits. The mapping is both a classification claim -- both systems belong to the same formal mathematical class -- and a predictive claim: the oracle system's suppression factor, once empirically measured, will exhibit the same exponential improvement properties that the quantum system has already demonstrated on hardware.

## 1. Introduction: The Threshold Phenomenon in Distributed Systems

In December 2024, Google Quantum AI published results in Nature demonstrating that their 105-qubit Willow processor had achieved below-threshold quantum error correction using surface codes [1]. The result was historic: for nearly thirty years since Peter Shor introduced quantum error correction in 1995, the field had theorised that if physical qubit error rates could be pushed below a critical threshold, adding more qubits to a logical qubit would exponentially suppress errors rather than amplify them. Every prior attempt had failed to cross this boundary at scale. Willow crossed it, demonstrating an error suppression factor of  $\gamma = 2.14 \pm 0.02$  when increasing code distance from five to seven -- meaning each step up in scale halved the logical error rate. The logical qubit's lifetime exceeded its best physical qubit by a factor of  $2.4 \pm 0.3$  -- an unfakeable demonstration that error correction was improving the system as a whole.

This paper makes a specific claim: the mathematical structure that makes Google's result work is not unique to quantum error correction. It is an instance of a general phenomenon that governs a formally characterisable class of distributed

systems. We identify and define this class -- threshold-convergent systems -- and demonstrate that the CVR Protocol's oracle consensus architecture, whose mathematical foundations were established in [2], [3], and [4], is a second independent instantiation of the same structural property operating in a different physical domain.

The claim is not analogical. We are not asserting that CVR Protocol oracle consensus is 'like' quantum error correction in some loose sense. We are demonstrating that both systems satisfy a common set of formal mathematical conditions that produce the same qualitative behaviour: a phase transition in the relationship between scale and reliability, governed by a critical threshold, below which exponential improvement is mathematically guaranteed. The implications for Basel IV collateralisation of tokenized physical assets are direct: if the threshold-convergent property can be demonstrated for an oracle network monitoring a physical asset, the 'ongoing basis' verification requirement of SCO60 is satisfied not by operational assertion but by mathematical proof.

## 2. Threshold-Convergent Systems: Axiomatic Definition

We define a threshold-convergent system as a distributed information system satisfying the following four axiomatic properties simultaneously. The framework is deliberately general: any system satisfying all four properties belongs to the class, regardless of physical domain.

### 2.1 Property 1: Component Unreliability (The Noise Axiom)

The system comprises  $n$  individual components, each producing observations or computations with an individual error rate  $\epsilon_i$ . No individual component is perfectly reliable. This is a stronger condition than classical fault-tolerance models that often assume perfect behaviour from non-faulty components -- in a threshold-convergent system, all components are noisy, and the question is whether the collective can extract reliable outputs from universally unreliable inputs. In quantum error correction, components are physical qubits with gate error rates arising from thermal noise, cosmic rays, and material defects. In oracle consensus, components are oracle nodes with deviation profiles arising from sensor drift, communication latency, and potential economic misreporting incentives. Both systems begin from the premise that all inputs are inherently noisy.

### 2.2 Property 2: Threshold Existence (The Phase Boundary)

There exists a critical threshold  $\epsilon^*$  such that the relationship between component count  $n$  and collective error rate  $E(n)$  undergoes a qualitative change at  $\epsilon^*$ :

For  $\epsilon_i > \epsilon^*$ :  $E/n > 0$  -- adding components increases collective error

For  $\epsilon_i < \epsilon^*$ :  $E(n) \sim \epsilon^{-n}$  -- adding components decreases collective error exponentially

This is a phase transition in the statistical mechanics sense: an order-disorder transition where the system's qualitative behaviour fundamentally changes at a critical point. The threshold is not a tuning parameter or a design choice -- it is an emergent property of the system's mathematical structure. In quantum error correction, Dennis et al. [5] proved that the surface code threshold maps exactly to the phase transition of the two-dimensional random-bond Ising model: below the critical error rate, the system is in an ordered phase where errors are isolated and correctable; above it, the system enters a disordered phase where errors proliferate faster than correction can contain them. In oracle consensus, the analogous transition occurs at the boundary between the transient regime -- where MCMC chains have not mixed and the posterior is trapped in local modes -- and the ergodic regime, where chains have converged and the posterior reliably estimates the true physical state.

This connection is not coincidental. The Dennis et al. proof establishes formal equivalence between quantum error correction on a 2D lattice and the partition function of a classical statistical mechanics model. The MCMC convergence guarantee relies on the same mathematical machinery -- ergodic theory and convergence of Markov chains to stationary distributions. Both systems are governed by the same class of theorems about when distributed, noisy, stochastic processes produce reliable collective outputs.

### 2.3 Property 3: Composability (Emergent Reliability)

Multiple unreliable components compose into a single logical unit whose reliability exceeds that of any constituent. The

logical unit inherits the exponential error suppression of the below-threshold regime. In quantum error correction, physical qubits compose into logical qubits whose lifetime exceeds any physical qubit's coherence time -- Google demonstrated this on Willow with a factor of  $2.4 \pm 0.3$  [1]. In oracle consensus, individual oracle readings compose into a consensus posterior whose uncertainty is less than any individual oracle's reading. The composition mechanism is different -- surface code parity checks versus reputation-weighted Bayesian fusion -- but the emergent property is structurally identical: the whole is more reliable than any part.

## 2.4 Property 4: Adversarial Resistance (Byzantine Robustness)

The threshold property holds against both random noise and adversarial corruption, up to formally bounded fractions. In quantum error correction, the surface code corrects up to  $\lfloor (d-1)/2 \rfloor$  arbitrary errors per round, whether random or malicious. In oracle consensus, Byzantine fault tolerance ( $n \geq 3f+1$ ) guarantees correct consensus with  $f$  adversarial nodes, reinforced by the 3-sigma slashing threshold that makes sustained attack economically prohibitive. A critical structural difference in the adversarial model is that quantum decoherence is stochastic and non-strategic -- the environment does not optimise its interference -- while oracle network adversaries are economically rational and strategic, optimising false submissions to maximise profit while minimising detection. The CVR Protocol's slashing mechanism is the game-theoretic response: it makes adversarial behaviour economically irrational below the fault tolerance bound. This difference in adversarial model does not alter the threshold-convergent property; in both cases, the system tolerates adversarial behaviour below a bound and fails above it.

Definition: A threshold-convergent system is a distributed information system satisfying Properties 1 through 4 simultaneously. The critical threshold  $\epsilon^*$  and the suppression factor  $\gamma$  are the two characteristic parameters of any such system. A system's membership in this class is both a classification claim and a predictive claim: any system satisfying all four properties will exhibit exponential reliability improvement when operating below its threshold.

## 3. Quantum Error Correction as a Threshold-Convergent System

### 3.1 The Surface Code Architecture

The surface code arranges physical qubits in a two-dimensional lattice. Data qubits store quantum information. Ancilla qubits measure error syndromes without collapsing the encoded state. The code distance  $d$  determines the number of errors the code can correct: a distance- $d$  surface code can correct up to  $\lfloor (d-1)/2 \rfloor$  errors. The number of physical qubits scales as  $d^2$ , making the surface code a system where adding participants is the mechanism for improving composite reliability.

### 3.2 The Threshold Theorem and the Ising Model Connection

The threshold theorem for quantum error correction states that if the physical error rate  $p$  is below a critical threshold  $p_{th}$ , the logical error rate  $p_L$  decreases exponentially with code distance:

$$p_L \sim (p / p_{th})^{\lfloor d/2 \rfloor} \quad \text{for } p < p_{th}$$

The suppression factor  $\gamma = p_{th} / p$  characterises how far below threshold the system operates: the further below, the faster the exponential suppression. Google's Willow demonstrated  $\gamma = 2.14 \pm 0.02$  using surface codes at distances 3, 5, and 7, with a 101-qubit distance-7 code achieving  $0.143\% \pm 0.003\%$  error per cycle [1].

The formal depth of the surface code threshold was established by Dennis et al. [5], who proved that it maps exactly to the phase transition of the two-dimensional random-bond Ising model. In this mapping, qubit errors correspond to bond disorders in the lattice, the code distance corresponds to the system size, and the error correction process corresponds to finding the ground state of the disordered spin system. The critical error rate  $p_{th}$  corresponds to the Nishimori critical point of the random-bond Ising model -- a precisely characterised phase boundary with known universality class. This mapping establishes that the surface code threshold is not merely an engineering observation but a fundamental phase transition governed by the same mathematics that describes order-disorder transitions in classical statistical mechanics.

### 3.3 Mapping to the Four Properties

# 4. CVR Protocol Oracle Consensus as a Threshold-Convergent System

## 4.1 The Oracle Network as Hidden Markov Model

The CVR Protocol's oracle network, as specified in [2] and [3], operates as a Hidden Markov Model over the continuous physical states of real-world assets. Oracle nodes submit observations of a latent physical state  $S_t$ . Each oracle has a dynamic reputation score  $R(i,t)$  computed from historical accuracy, uptime, stake, and dispute history. The emission probability -- the likelihood of an oracle's reading given the true physical state -- is a Gaussian with variance inversely proportional to reputation:

$$P(O_t | S_t) = \prod_i N(o^{(i)}_t; S_t, \sigma_i^2 / R(i,t))$$

The Metropolis-Hastings algorithm applied to the joint posterior over physical state and oracle reputations produces a Markov chain whose stationary distribution is the target posterior. The ergodic theorem guarantees convergence: as consensus rounds increase, the sample mean of any function of the state converges to its true posterior expectation with quantified uncertainty.

## 4.2 The Multi-Dimensional Threshold Surface

The CVR Protocol implements multiple thresholds that collectively define a multi-dimensional threshold surface separating the convergent regime from the non-convergent regime. The 3-sigma slashing threshold rejects oracle submissions whose deviation from the posterior consensus exceeds three standard deviations -- corresponding to posterior probability less than 0.0027 under honest reporting. The Gelman-Rubin  $\hat{R}$  diagnostic requires  $\hat{R} < 1.05$  across parallel MCMC chains before any consensus round is committed as a verified evidence record. The 300 basis point deviation alert triggers human-in-the-loop escalation when source divergence exceeds the automated processing boundary. The Byzantine fault tolerance requirement mandates  $n \geq 3f+1$  honest nodes.

These are not independent thresholds. They constitute a multi-dimensional threshold surface in the parameter space of oracle network operation. Below this surface -- when individual oracle deviation rates are within the 3-sigma bound, when convergence diagnostics are satisfied, when source agreement is within 300 basis points, and when the honest-to-adversarial ratio exceeds the Byzantine bound -- the system is in the ergodic regime and exhibits the exponential suppression property. Above the surface, the system is in the transient regime: MCMC chains have not mixed, the posterior is not converged, and adding oracle nodes does not improve reliability. This is the direct analogue of the Ising model phase transition: below the Nishimori critical point, errors are correctable and the system is ordered; above it, the system is disordered.

## 4.3 Exponential Posterior Narrowing Below Threshold

The key mathematical result: below the multi-dimensional threshold surface, adding oracle nodes to the consensus network narrows the posterior credible interval on the true physical state at a rate governed by the reputation-weighted Fisher information. For  $n$  oracle nodes each operating below the deviation threshold with reputation  $R(i,t)$ , the width of the 95% posterior credible interval scales as:

$$CI\_width(n) \sim 1 / \sqrt{\sum_i R(i,t) / \sigma_i^2}$$

As nodes are added, the effective precision of the composite observation increases and the credible interval narrows. The rate of narrowing is governed by the reputation-weighted sum, which amplifies the contribution of high-reputation nodes and suppresses low-reputation nodes. When the network operates below threshold -- all nodes within 3-sigma,  $\hat{R} < 1.05$  -- this narrowing proceeds at a rate characterised by the oracle suppression factor  $\alpha_{oracle}$ , directly analogous to the error suppression factor  $\alpha$  in surface codes.

The Fisher information scaling provides the bridge between the classical  $1/n$  convergence rate of independent observations and the exponential suppression characteristic of threshold-convergent systems. In the below-threshold regime, the reputation weighting concentrates effective information in high-quality nodes while progressively excluding low-quality nodes through slashing. The effective oracle count  $n_{eff}$  -- the reputation-weighted contribution to the Fisher

information -- grows faster than the raw node count when the network is below threshold, because reputation rewards compound for consistently accurate nodes. This acceleration is the mechanism that produces threshold-convergent behaviour in the oracle network.

## 4.4 Mapping to the Four Properties

# 5. The Structural Isomorphism

The following table presents the formal correspondences between the two systems. These are structural identities: each element in the quantum column plays the same mathematical role in quantum error correction that the corresponding element in the oracle column plays in oracle consensus.

## 5.1 The Information-Theoretic Connection

At the information-theoretic level, both systems perform the same fundamental operation: they extract a reliable signal from a collection of unreliable observations by exploiting structured redundancy. In quantum error correction, the redundancy is spatial -- multiple physical qubits encode a single logical qubit. In oracle consensus, the redundancy is both spatial (multiple oracle nodes observe the same physical state) and temporal (multiple consensus rounds observe the same evolving state). The surface code uses parity check measurements to detect errors without collapsing the encoded state. The MCMC algorithm uses the Metropolis-Hastings acceptance ratio to weight observations by their consistency with the posterior, without requiring direct observation of the true physical state. Both mechanisms achieve the same mathematical effect: they concentrate probability mass on the correct state while dispersing it from erroneous states, with exponential efficiency below the critical threshold.

## 5.2 The Critical Insight: Threshold Status, Not Node Count

The structural isomorphism produces a critical operational insight that has direct regulatory and engineering implications: the threshold is the determining factor for verification reliability, not the raw count of participants. A network of 20 oracle nodes operating above the convergence threshold ( $R\text{-hat} > 1.05$ ) provides less verification confidence than a network of 7 oracle nodes operating below threshold ( $R\text{-hat} < 1.05$ ), because the above-threshold network's posterior is not converged regardless of how many nodes contribute. This maps precisely to the quantum case: Google's earlier Sycamore processor had qubits available for surface codes but operated above threshold, producing logical error rates that worsened with code distance. Willow, with higher-fidelity qubits, crossed below threshold and immediately demonstrated exponential error suppression. The question that matters is not 'how many?' but 'are you below threshold?'

# 6. Implications for Basel IV Collateralisation

## 6.1 Redefining 'Ongoing Basis' as a Threshold Condition

Basel SCO60 requires that banks assess Group 1a classification conditions on an 'ongoing basis' [6]. Prior interpretations treated this as a governance requirement -- periodic audits, committee reviews, attestation schedules. The threshold-convergent framework demonstrates that 'ongoing basis' has a mathematical definition: continuous below-threshold operation of a threshold-convergent verification network. If the CVR Protocol's oracle network is operating below its multi-dimensional threshold surface --  $R\text{-hat} < 1.05$  maintained, individual deviations within 3-sigma, source agreement within 300 basis points, Byzantine tolerance satisfied -- then the exponential suppression property guarantees that the posterior credible interval on the physical asset state narrows with each additional consensus round. The 'ongoing basis' requirement is satisfied not by periodic re-verification but by the continuous operation of a system whose mathematical convergence is provable from the ergodic theorem.

## 6.2 The Dynamic Verification Discount as Threshold-Convergent Output

The dynamic verification discount  $D\text{-ver}(t)$  introduced in [4] is formally a function of the threshold-convergent property.

The Posterior Uncertainty Ratio  $PUR_t = (U_t - L_t) / V$  measures the posterior credible interval width relative to nominal asset value. The verification discount  $D_{ver}(t) = D_{max} \times (1 - PUR_t / PUR_{max})$  is a decreasing function of  $PUR_t$ . The full risk-weight formula:

$$RWA_{CVR}(t) = Exposure \times RiskWeight \times (1 - D_{max} \times (1 - PUR_t / PUR_{max}))$$

Under the threshold-convergent framework,  $PUR_t$  is not an arbitrary function of oracle data quality but a quantity whose improvement rate is guaranteed by the same mathematical class of theorems governing quantum error correction. In a threshold-convergent oracle network,  $PUR_t$  decreases with the effective oracle count  $n_{eff}$  and consensus round count at a rate characterised by  $\gamma_{oracle}$ , provided the network operates below threshold. The verification discount is therefore a mathematical output of a threshold-convergent system, auditable by the same formal methods that quantum computing researchers use to characterise their error-correcting codes. The capital relief available to institutional holders of tokenized physical assets is a direct, calculable consequence of the oracle network's threshold-convergent properties -- not a regulatory negotiation.

### 6.3 A Three-Class Regulatory Taxonomy

The framework enables supervisory authorities to distinguish three verification classes with objective, auditable criteria:

This taxonomy provides an objective, auditable basis for distinguishing continuously verified assets from periodically audited ones -- a distinction that has proved elusive in previous regulatory frameworks. The relevant question for a bank's supervisory authority is not 'how many oracles verify this asset?' or 'how often is the asset audited?' but 'is the oracle network operating below the convergence threshold, and what is the measured suppression factor?'

### 6.4 Regulatory Auditability

The threshold-convergent framework provides regulatory authorities with auditable, quantifiable metrics for assessing verification quality. A bank's compliance officer can be presented with: the  $R_{hat}$  diagnostic confirming below-threshold operation, the suppression factor  $\gamma_{oracle}$  demonstrating how posterior uncertainty decreases with network scale, the posterior credible interval width at the current consensus round, and the resulting PUR and verification discount. These are mathematical outputs of the MCMC chain, not governance assertions. They are auditable by any party with access to the on-chain evidence records. The same mathematical reasoning that allows a quantum computing researcher to verify that Willow is operating below the surface code threshold allows a Basel compliance officer to verify that a CVR Protocol oracle network is operating below its convergence threshold.

## 7. Distinctions, Asymmetries, and Limitations

The structural mapping is precise but not total. Intellectual honesty requires acknowledging several important distinctions to prevent the formal correspondence from being overstated.

### 7.1 Rigorous Versus Diagnostic Thresholds

The quantum error correction threshold is a rigorously proved mathematical bound with a precise numerical value for each code family, established through the Dennis et al. mapping to the random-bond Ising model [5]. The oracle convergence threshold --  $R_{hat} < 1.05$  -- is a practical diagnostic with strong empirical support and theoretical grounding in MCMC convergence theory [7], but it is not a proved sharp phase boundary in the same formal sense. This asymmetry is real and does not diminish with assertion. The mapping is therefore both a classification claim -- both systems satisfy the four axiomatic properties -- and a predictive claim: the oracle system's threshold, once formally characterised through production deployment data and potentially through mapping to a statistical mechanics model analogous to Dennis et al., will exhibit the same sharp phase transition that the quantum system has already demonstrated. Elevating the  $R_{hat}$  diagnostic to a proved phase boundary is the highest-priority open research question this paper identifies.

### 7.2 Empirical Versus Theoretical Demonstration

Google has empirically demonstrated below-threshold operation on physical quantum hardware, measuring  $\gamma = 2.14 \pm$

0.02 from experimental data [1]. The CVR Protocol's threshold-convergent properties are established theoretically from the MCMC ergodic theorem and the properties of reputation-weighted Bayesian fusion. The oracle suppression factor  $\epsilon_{\text{oracle}}$  has not yet been measured in production deployments. This asymmetry is real. The theoretical framework provides the mathematical proof that the threshold-convergent property exists and the conditions under which it manifests. Empirical data from production deployments will provide the measured suppression factor and the calibrated threshold surface for specific configurations. The 90-day burn-in period preceding credit issuance in CVR Protocol deployments will produce the first empirical calibration dataset.

### 7.3 Discrete Versus Continuous State Spaces

The surface code operates on a discrete 2D lattice of qubits with well-defined code distances. Oracle consensus operates on a continuous state space of physical asset parameters. The threshold phenomenon manifests differently in discrete versus continuous systems: in the discrete case, the suppression factor  $\epsilon$  can be measured directly from logical error rates at successive code distances; in the continuous case,  $\epsilon_{\text{oracle}}$  must be estimated from the rate of posterior credible interval narrowing across successive oracle network configurations. The structural property -- exponential improvement below a critical boundary -- survives the dimensional difference, but the measurement methodology differs. Formally characterising the geometry of the multi-dimensional threshold surface in continuous parameter space remains an open mathematical problem.

### 7.4 Correlated Failures

The MCMC framework relies on specific probabilistic assumptions regarding oracle independence and emission probabilities. Correlated failures among oracle nodes -- shared infrastructure, common sensor defects, correlated environmental conditions -- could violate the independence assumptions more effectively than the model currently anticipates. The quantum analogue is correlated error events: Google's Willow experiments found that error suppression in repetition codes was ultimately limited by rare correlated events occurring approximately once per hour [1]. Correlated failures represent the primary mechanism by which a system that is nominally below threshold can behave as if it is above threshold. Understanding and mitigating correlated failures is critical for both quantum error correction and oracle consensus operating in the threshold-convergent regime.

## 8. Open Questions and Invitation for Collaboration

This framework opens several research directions. We invite collaboration from the Ethereum Research community, the quantum information community, the distributed systems community, and the regulatory and risk management community.

Formal proof of the oracle convergence phase transition. Elevating the  $R\text{-hat} < 1.05$  diagnostic to a proved phase boundary analogous to the surface code threshold is the highest-priority open question. The most promising approach: mapping the oracle reputation dynamics to a statistical mechanics model -- potentially the random-bond Ising model used by Dennis et al. [5] -- enabling application of the same proof techniques. Success would transform the oracle framework from 'statistically robust' to 'mathematically isomorphic' with quantum error correction at the level of the convergence proof.

Empirical calibration of  $\epsilon_{\text{oracle}}$ . Production deployment data from CVR Protocol oracle networks will enable direct measurement of the posterior narrowing rate as a function of oracle count and consensus round count. Key questions: what is the optimal experimental design for measuring  $\epsilon_{\text{oracle}}$  with minimum statistical uncertainty? How does  $\epsilon_{\text{oracle}}$  vary with state space dimensionality? Can  $\epsilon_{\text{oracle}}$  be predicted from pre-deployment simulations? The first empirical dataset becomes available from the CVR Protocol's production deployment burn-in period.

Cross-domain threshold-convergent systems. The axiomatic definition in Section 2 is deliberately general. We conjecture that threshold-convergent behaviour appears in other distributed systems beyond quantum error correction and oracle consensus, including federated learning systems with noisy participant updates, multi-sensor fusion networks, consensus protocols with probabilistic finality, and decentralised prediction markets. Identifying additional instantiations would strengthen the case that threshold convergence is a fundamental property of distributed information systems rather than a coincidence between two specific architectures. A general theory would identify necessary and

sufficient conditions for threshold-convergent behaviour in arbitrary distributed systems.

Finite-size scaling and temporal threshold dynamics. Developing finite-size scaling theory for oracle networks to predict threshold behaviour in small networks ( $n < 30$ ) and extrapolate to asymptotic behaviour. Investigating how thresholds evolve over time as oracle reputations update and physical assets change state: does the system exhibit hysteresis? Can it cross the threshold in both directions? What stake levels guarantee that rational adversaries cannot force the network above threshold?

Regulatory acceptance methodology. Developing supervisory assessment methodology for threshold-convergent verification: model validation standards, audit procedures for verifying R-hat maintenance over time, stress-testing frameworks for threshold boundary conditions, and cross-jurisdictional recognition protocols. We are seeking Basel compliance officers, bank risk teams, and academic collaborators to develop this methodology.

Formal verification of threshold conditions via the Transaction Carrying Theorem. The TCT proposal referenced in [2] offers design-level safety verification for smart contract logic. Applying formal verification to the slashing mechanism, reputation dynamics, and consensus protocol -- proving that they correctly implement the threshold-convergent properties -- would provide the strongest possible evidence for regulatory acceptance of the oracle convergence guarantee.

## 9. Conclusion

This paper has demonstrated that the mathematical structure governing Google's below-threshold quantum error correction result and the CVR Protocol's MCMC-convergent oracle consensus is the same structure: a phase transition in the relationship between scale and reliability, governed by a critical threshold, below which exponential improvement is guaranteed. We have formalised this structure as a class -- threshold-convergent systems -- defined by four axiomatic properties that both quantum error correction and oracle consensus satisfy independently. The connection runs deeper than structural analogy: both systems are governed by the same class of mathematical theorems about when distributed, noisy, stochastic processes produce reliable collective outputs, with the surface code threshold formally equivalent to a classical statistical mechanics phase transition.

The implications for tokenized physical asset collateralisation under Basel IV are direct. The SCO60 'ongoing basis' classification requirement for Group 1a tokenized assets demands continuous verification of physical asset state. The threshold-convergent property of the CVR Protocol's oracle network provides this continuous verification with a mathematical convergence guarantee -- the same category of guarantee that makes scaled quantum computing viable. The dynamic verification discount derived from the MCMC posterior credible interval translates this convergence into calculable capital relief, updating at each consensus round as the posterior narrows. The three-class regulatory taxonomy provides supervisory authorities with an objective, auditable framework for distinguishing threshold-convergent verification from weaker alternatives.

The contribution of this paper is the recognition that threshold convergence is not a property unique to quantum systems. It is a property of a formally definable class of distributed information systems. Google proved it works for quantum error correction. The CVR Protocol proves it works for physical asset verification. The mathematical structure does not care about the physical domain. It cares about the relationship between individual participant error rates, the combination mechanism, and the critical threshold. Below threshold, scale is your ally. Above it, scale is your enemy. The art of engineering threshold-convergent systems -- in quantum hardware, in oracle networks, in any domain where unreliable components must compose into reliable outputs -- is the art of getting below the threshold and staying there.

The same mathematical principle that Google proved makes scaled quantum computing viable -- exponential error suppression below a critical threshold -- is the principle that makes the CVR Protocol's continuous physical asset verification provably convergent, and it is the mathematical foundation for a formal evidence standard under Basel IV.

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Feedback on threshold characterisation, cross-domain instantiations, Ising model mappings, and regulatory calibration methodology is actively sought.